## Week 15 notes

Reading: This website regarding interest rates - https://money.howstuffworks.com/interest-rate.htm

Why do we have money?
Economists mostly describe money as having 3 purposes:

1. Store of value - trade money for goods and services in the future
2. Unit of account - yardstick to measure economic transactions (prices are quoted in dollars, not "foregone ham")
3. Medium of exchange - what we use to buy goods and services; without it, we would have a barter system where you have to trade your goods and services for other goods and services (requires double coincidence of wants as we saw in the trade game early in the semester)

Types of money: commodity money vs fiat money
Commodity money - other intrinsic value to what is used for money (such as gold, which can be used in jewelry or dental fillings)

Fiat money - it is money because it is declared to be money by gov't, and people trust its common acceptance (such as our dollars)

Properties of ideal money (that our current money has or at least mostly has)

- Durable
- Portable
- Divisible
- Uniform quality
- Low opportunity cost to make it
- Relatively stable value over time
- Anonymity
- Hard to fake/counterfeit
- Easy, reliable means of transferring ownership

A future property of money that would be a substantial improvement would be easy, reliable proof of ownership (while somehow maintaining anonymity and easy, reliable means of transferring ownership). Think about what happens today if you drop a $\$ 20$ bill in the street. Nobody is returning that money to you. But imagine if you truly "owned" that bill, and nobody else could spend it until you've transferred the ownership. They have no incentive to keep it and thus you'd be more likely to actually get it back. Bitcoin and other new types of money like it make significant progress on this front.

Two-period consumption model

Remember when we talked about consumers deciding what to buy? We said it was based on three things: preferences, income, and prices. In two dimensions, we could capture that graphically using a budget line and an IC by solving the consumer problem.

$$
\begin{array}{ll}
\text { Budget constraint: } & Y=P_{1} Q_{1}+P_{2} Q_{2} \\
\text { where } & Y=\text { income } \\
& P_{1}=\text { price of good } 1 \\
& Q_{1}=\text { quantity of good } 1 \text { that you buy } \\
& P_{2}=\text { price of good } 2 \\
& Q_{2}=\text { quantity of good } 2 \text { that you buy }
\end{array}
$$

With a little bit of algebra, the budget constraint can become $Q_{2}=\frac{Y}{P_{2}}-\frac{P_{1}}{P_{2}} Q_{1}$


We can only show two goods when drawn on the board on paper. In reality, a consumer spends his income $Y$ buying many goods, so $Y=P_{1} Q_{1}+P_{2} Q_{2}+P_{3} Q_{3}+P_{4} Q_{4}+\ldots+P_{N} Q_{N}$ for all $N$ goods and services that exist in the world (millions and millions of them). What we choose as good 1 and good 2 (usually in
class it was corn and wheat, but any choice works) is really just a more concrete version of an abstract choice between any two "goods." So saying "corn" is more concrete than saying "all the goods you consumed last year." But in the end, we can define our goods however we'd like, provided that we update the budget constraint and the other good accordingly, along with our understanding of what the IC is representing and what the prices are.

So let's model a consumer making choices about consumption over two time periods. You can think about the time periods in a simple way (yesterday and today, this year and next year) or, if you're so inclined to think of it more like an economist would, draw a relevant model for consumer behavior (e.g. period 1 is working years and period 2 is retirement). The key is just to make sure you define your goods and budget in a way that makes sense. It would be silly to model yesterday's consumption vs today's consumption using your yearly salary from 2011. The "goods" under consideration must match the model.

So let's make it a bit more specific for this context and model a consumer making consumption choices across 2 years. Define $\mathrm{C}_{1}$ as total consumption in year 1 (including both quantity and price). So $\mathrm{C}_{1}$ itself is the total amount of money a consumer spent on food, rent, clothing, entertainment, transportation, medicine, etc. in year 1. In other words, $\mathrm{C}_{1}$ is the whole right hand side of the equation at the top of this page over an entire year. Define $C_{2}$ in the same way for year 2. The consumer makes $Y_{1}$ income the first year and $Y_{2}$ income the second year (there is no need to have $Y_{1}=Y_{2}$ ).

The point $\left(Y_{1}, Y_{2}\right)$ is obviously one possibility for $\left(C_{1}, C_{2}\right)$. In other words, the consumer spends his year 1 income ( $Y_{1}$ ) buying goods and services in year $1\left(C_{1}\right)$. In fact, he spends all of his $Y_{1}$ on $C_{1}$ because he wants to make himself as happy as possible, and, absent a system for saving or borrowing, he would make himself happiest by spending all of his income $Y_{1}$ to make $C_{1}$ as high as possible. Similarly, he would make $\mathrm{C}_{2}=\mathrm{Y}_{2}$.


But we know this situation is not realistic! Sometimes people save money and don't spend everything in the time period when they earned it, and sometimes they borrow money.

Question: Given that having more stuff makes consumers happier, why would someone save money?
Answer: Saving money now means you have more money to spend later. In our example, when there is no mechanism for transferring income across time periods, you spend all of $Y_{1}$ on $C_{1}$. But what if you could save? And what if you knew you needed a new car and a vacation next year? In order to make $C_{2}$ higher than $Y_{2}$ (in order to afford that car and vacation), you would save money this year to put toward next year's consumption. In other words, make $C_{1}$ LESS than $Y_{1}$ and "bank" that money in order to make $\mathrm{C}_{2}$ MORE than $\mathrm{Y}_{2}$. So rather than seeing saving as simply decreasing current consumption, realize that it allows the consumer to increase future consumption, too!

Interest rate - expressed as percentage of principal for one year, this is the additional amount paid by a borrower for the use of money that they borrow from a lender

## Principal - amount of money borrowed

Why do interest rates exist? You can read more about it in the website reading listed at the top of page 1, but the most succinct way of explaining why interest rates exist is that they are a way of incentivizing the lender to actually loan out money to the borrower. During the interim period before the loan is paid back (and as a hedge in case some loans aren't paid back), the lender does not have access to his money. The only way to encourage someone to forgo their own money for a period is to agree to pay them back more once the debt is repaid. The interest rate represents the amount of "more" pay.

So let's update our model now to include interest rates and saving/borrowing. Our two years are the only periods under consideration, and the consumer cannot have debt or savings at the end of year 2. But there is a full market for borrowing or saving as much as you want and can in year 1 , and then either dipping into year 2 income or adding to it for the second period. Let the interest rate be $r$. It is the same here for borrowers and lenders. ${ }^{1}$

With an interest rate of $r$, if I save $\$ 1$ in year 1 , then I have $\$ 1(1+r)$ to spend extra beyond $Y_{2}$ in year 2 . So if $r=5 \%=0.05$, then saving $\$ 1$ in year 1 (meaning $C_{1}$ is $\$ 1$ less than $Y_{1}$ ) yields an extra $\$ 1.05$ in year 2.

Similarly, the model allows borrowing in year 1 so that $C_{1}>Y_{1}$. How much do you have to pay back though? Well, again, consider the interest rate. If you borrow $\$ 1$ to spend today (to make $\mathrm{C}_{1} \$ 1$ more than $Y_{1}$ ) and $r=5 \%$, you'll have to pay back $\$ 1.05$ next year (and thus make $C_{2} \$ 1.05$ less than $Y_{2}$ ).

Given what we have so far $-Y_{1}, Y_{2}, C_{1}, C_{2}$, and $r$ - how can we model the consumer behavior graphically?
Answer: We know that $\left(Y_{1}, Y_{2}\right)$ has to be a point along the two-period budget constraint since it represents no borrowing and no lending. What other points can we solve for? Let's focus on the endpoint on the $C_{2}$ axis. That means that $C_{1}=0$, so you spend nothing in period 1 and save your entire year 1 income, $Y_{1}$. Then in $C_{2}$ you get to spend all of $Y_{2}$, plus all of the saved $Y_{1}$, plus all of the interest earned on $Y_{1}$. In other words, $C_{2}=Y_{2}+Y_{1}(1+r) .{ }^{2}$ Likewise, we can focus on the endpoint on the $C_{1}$ axis.

[^0]Here, $C_{2}=0$, so all your money is spent in period 1. But the interest rate bites a bit. You are not able to spend all of $Y_{2}$ in period 1, but rather just $Y_{2} /(1+r)$ of it. ${ }^{3}$ So the $C_{1}$ axis endpoint is $C_{1}=Y_{1}+Y_{2} /(1+r)$.

Connect those 3 points (the two endpoints and the ( $Y_{1}, Y_{2}$ ) point) and we have a complete budget constraint for our 2-period model! We can borrow any amount between 0 and $Y_{2} /(1+r)$ or save any amount between 0 and $Y_{1}$. Once we know $Y_{1}, Y_{2}, r$, and $C_{1}$, we can always solve for $C_{2}$ ! (Or, rather, once we know any of those four variables, we can always solve for the fifth one.)

What is $\mathrm{C}_{2}$ ?

$$
\begin{gathered}
C_{2}=Y_{2}+\left(Y_{1}-C_{1}\right)(1+r) \\
C_{2}=Y_{2}+Y_{1}(1+r)-C_{1}(1+r)
\end{gathered}
$$

Keep in mind how that equation looks like the slope-intercept formula from algebra class


[^1]Question: So what point ( $\mathrm{C}_{1}, \mathrm{C}_{2}$ ) does a consumer ultimately choose?
Answer: Think about what we have so far. We've incorporated income ( $\mathrm{Y}_{1}$ and $\mathrm{Y}_{2}$ ). The interest rate is nothing more than the price of $C_{1}$ above $Y_{1}$ (or "extra income" you receive in year 2 if $C_{1}$ is less than $Y_{1}$ ). We still need to include preferences, which can be shown with ICs.

Ultimately, the choice of ( $\mathrm{C}_{1}, \mathrm{C}_{2}$ ) will depend on the preferences for consumption over the two periods. Some people (college students, new small business owners) are borrowers: they "use" some $Y_{2}$ in order to consume more $C_{1}$ now than their meager $Y_{1}$ would allow. Still others borrow in order to buy a house (mortgage is just a fancy name for "house loan") or a car. In order for someone to borrow now, someone else must be willing to decrease their $\mathrm{C}_{1}$ below $\mathrm{Y}_{1}$ and offer the other person that money. The lender would then receive back the borrowed amount, plus interest, in period 2.

So who are lenders in society? Even though banks do the lending, the lenders themselves are anyone with a savings account at the bank.

As we did with the consumer problem in the first part of the course, let's combine income, prices, and preferences to show the actual choice of consumption. We do this in the same manner as before - push the IC out as far as possible until it is tangent to the budget constraint. Depending on the location of the IC, which is simply a reflection of personal preferences, our consumers could be borrowers or savers.



What happens if $r$ changes? Let's walk through the effects, how we can show it graphically, and then discuss who is hurt or helped by changes in the interest rate ("who" being borrowers or lenders).

- $\quad r$ increases - This situation is good for lenders (savers) and bad for borrowers. Why? Because with a higher interest rate, more interest is earned on every dollar saved today. Similarly, it costs more interest to pay back $\$ 1$ borrowed today if $r$ is higher.

Old rate: 5\%
Lender: Lent \$1 today, gets \$1.05 next period
Borrower: Borrowed \$1 today, pays back \$1.05 next period

New rate: 8\% (went up)
Lender: Lent \$1 today, gets \$1.08 next period
Borrower: Borrowed \$1 today, pays back \$1.08 next period

How does the graph change? Both endpoints have $r$, so both endpoints change when $r$ increases; the endpoint on the $C_{1}$ axis moves left and the endpoint on the $C_{2}$ axis moves up. The slope is $-(1+r)$, so when $r$ increases, the slope gets steeper. The only point that remains the same is $\left(\mathbf{Y}_{1}, \mathbf{Y}_{\mathbf{2}}\right)$. Why? Because if you aren't borrowing or lending, a change in the interest rate doesn't affect you at all!

Notice what has happened to the budget constraint in the area to the right of $\left(Y_{1}, Y_{2}\right)$, i.e. where borrowers reside. The new budget constraint is closer to the origin. Clearly, a borrower will have to move to a lower IC, making him worse off.

For lenders, their section of the budget constraint is further out from the origin, so they move to a higher IC.

## SEE NEXT TWO PAGES FOR GRAPHICAL SOLUTION

- ON YOUR OWN - Show what happens to the budget constraint when $r$ decreases. What happens to the slope of the budget constraint? What happens to the endpoints? Who is hurt by this, borrowers or lenders? Who is helped by the interest rate change, borrowers or lenders?
(Hopefully, you realize the answers to all of those questions are the opposite of what we said above, where $r$ increases. But convince yourself of it by drawing the graph and showing an interest rate decrease in this ON YOUR OWN exercise.)


$$
r_{2}>r_{1}
$$

notice that new (red) budget line is steeper fut still goes through $\left(y_{1}, y_{2}\right)$



[^0]:    ${ }^{1}$ A more realistic situation would have a higher rate for borrowers and a lower rate for lenders, with banks acting as the intermediary institution between the sides. That may be addressed in a homework assignment question. ${ }^{2}$ Remember that there is no $3^{\text {rd }}$ year or beyond here. So there's no benefit to saving money in year 2.

[^1]:    ${ }^{3}$ It should be obvious why that is. If you multiply $Y_{2} /(1+r)$ by (1+r), you get $Y_{2}$. In other words, borrowing $Y_{2} /(1+r)$ in the first period is the most you can borrow in order to be able to afford to pay back that principal plus the earned interest in period 2 , where you will have only $Y_{2}$. If you borrowed $Y_{2}$ in period 1 , you would owe back $Y_{2}(1+r)$, and you would be unable to afford paying that back.

